The rheological model (1) is a generalization of several well-known models. For example, if the yield stress τ_0 vanishes, Eq. (1) reduces to a rheological power-law equation. When m = n (τ_0 = 0) we obtain an ordinary Newtonian fluid. The value of Re_{cr} for τ_0 = 0 and m = n reduces to the value for a Newtonian fluid [2] (Fig. 4). Our results for Re_{cr} in the case τ = 0 (Fig. 4) reduce to those for a non-Newtonian fluid with a rheological power law [6].

NOTATION

u_i, velocity vector components; τ_0, yield stress; μ, shear viscosity; m, n, rheological constants; U, velocity of the unperturbed flow; α, wave number; L, channel width; ρ, density; φ, perturbation amplitude; c, phase velocity; U_0, characteristic velocity; $H_{1/3}^{(1,2)}$, Hankel functions of the first and second kind of order 1/3; τ, Reynolds stress.

LITERATURE CITED

1. Z. P. Shul'man, Convective Heat and Mass Transfer of Rheologically Complex Fluids [in Russian], Énergiya, Moscow (1975).
2. Tsia Tsao Lin, Theory of Hydrodynamic Stability [Russian translation], IL, Moscow (1958).
3. N. V. Mikhailov and P. A. Rebinder, "On the structural mechanical properties of dispersive and high-molecular systems," Kolloidn. Zh., No. 17, 107-119 (1955).
4. W. Wasow, Asymptotic Expansions for Ordinary Differential Equations, Kreiger (1976).
5. L. K. Martinson, K. B. Pavlov, and S. L. Simkhovich, "Hydrodynamic stability of the gradient flow of a conducting fluid with a rheological power law in a transverse magnetic field," Magnitn. Gidrodin., No. 2, 35-40 (1972).
6. A. M. Makarov, L. K. Martinson, and K. B. Pavlov, "Stability of a non-Newtonian fluid with a rheological power law under plane flow," Inzh.-Fiz. Zh., 16, 793 (1969).

HYDRODYNAMIC INSTABILITY OF THE AXISYMMETRIC FLOW OF AN IDEAL FLUID WITH AN INTERPHASE

V. E. Epikhin and V. Ya. Shkadov

UDC 532.5.013.4

We study the instability under simultaneous rotational and translational flow of a fluid and ambient medium in the cases of a cylindrical annular jet, capillary jet, and cylindrical fluid layers on the inner and outer surfaces of a cylinder.

The type of flow under study is schematically illustrated in Fig. 1. Reviews of the literature and some new experimental results on instabilities of jets can be found in [1-9]. The stability of a fluid on a rotating cylindrical surface was studied in [10, 11]. In the present paper the stability of potential flow is considered in the most general formulation. Such flows are used in vaporizers, heat-transfer devices, chemical reactors, in the paper-pulp industry, and also in vertical–centrifugal methods of producing mineral fibers [11].

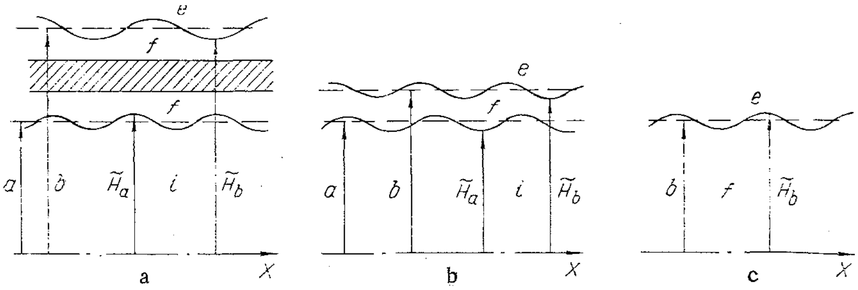
In a cylindrical coordinate system with the axis of coordinates taken along the symmetry axis of the problem, the flow is described by the potential functions

$$\Phi_f^0(X,\ \theta) = U_f X + \Gamma_f \theta,\quad \Phi_{e,i}^0 = U_{e,i} X + \Gamma_{e,i}\,\theta. \tag{1}$$

where the second term in both equations gives the velocity potentials of line vortices along the axis of rotation with circulations $2\pi\Gamma_f$, $2\pi\Gamma_e$, $2\pi\Gamma_i$, respectively [1]. At t = 0 a potential wave perturbation of infinitesimal amplitude is applied to the unperturbed flow. The potential functions of nonsteady motion satisfy the Laplace equation and the Cauchy–Lagrange integral in a flow region to be determined as part of the solution. The boundary conditions express the jump in the normal stress due to surface tension, the continuity of streamline flow at the boundaries, and the boundedness of the potentials on the axis and at infinity, and also the periodicity

M. V. Lomonosov Moscow State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 45, No. 1, pp. 64-72, July, 1983. Original article submitted February 4, 1982.

0022-0841/83/4501-0761$07.50



a b c

Fig. 1. Schematic diagram of the flows under study: (a) fluid layer on the surface of a rotating cylinder; (b) annular jet; (c) capillary jet.

of flow along the axis. In the linear approximation the problem in each of the flow regions is written in the form

$$\Delta\tilde{\Phi} = 0,\ \tilde{\Phi}_t + U\tilde{\Phi}_X + \frac{\Gamma}{Y}\left(\frac{\tilde{\Phi}_\theta}{Y} - \frac{\Gamma\tilde{H}}{Y^2}\right) = -\tilde{p}, \tag{2}$$

$$\tilde{H}_t + U\tilde{H}_X \frac{\Gamma}{Y^2}\tilde{H}_\theta = \tilde{\Phi}_Y \quad (Y = a,\ b), \tag{3}$$

$$\tilde{\Phi}_Y = 0, \quad Y = 1; \tag{4}$$

$$\rho_i\tilde{p}_i - \tilde{p}_a = -\frac{W_a}{a^2}(\tilde{H}_a + a^2\tilde{H}_{aXX} + \tilde{H}_{a\theta\theta}), \tag{5}$$

$$\frac{\tilde{H}_{a\theta}}{a^2}(\Gamma_f - \Gamma_i) + \tilde{H}_{aX}(U_f - U_i) = \tilde{\Phi}_{fY} - \tilde{\Phi}_{iY} \quad (Y = a),$$
$$\tilde{p}_b - \rho_e\tilde{p}_e = -\frac{W_b}{b^2}(\tilde{H}_b + b^2\tilde{H}_{bXX} + \tilde{H}_{b\theta\theta}), \tag{6}$$

$$\frac{\tilde{H}_{b\theta}}{b^2}(\Gamma_f - \Gamma_e) + \tilde{H}_{bX}(U_f - U_e) = \tilde{\Phi}_{fY} - \tilde{\Phi}_{eY} \quad (Y = b),$$
$$\tilde{\Phi}_i < \infty, \quad Y \to 0, \tag{7}$$

$$\tilde{\Phi}_e < \infty, \quad Y \to \infty. \tag{8}$$

These equations and boundary conditions are in dimensionless form using the following scales: the characteristic length R_0 is taken as the mean radius of the initial cross-section of the annular jet, the initial radius of the capillary jet, or the inner or outer radius of the surface of the cylinder. We also use the reference velocity U_0 and the density of the fluid ρ_F. Let $\lambda = 2\pi/\alpha$ be the wavelength, where α is the wave number of the initial perturbation. The solution of the homogeneous problem (2), (3), (5)-(8) for the capillary jet, (2)-(5), (7) for a fluid layer on the inner cylinder surface, and (2)-(4), (6), (8) for a layer on the outer surface will be sought in the form of traveling waves

$$\tilde{R}(X,\ Y,\ \theta,\ t) = R_*(Y)\,E,\ \ \tilde{H}(X,\ \theta,\ t) = H_*E,\ E = \exp[i(\alpha X + \beta\theta - Ct)].$$

Here $\tilde{R}$, $\tilde{H}$ denote one of the quantities $\tilde{\Phi}$, $\tilde{p}$, $\tilde{H}_a$, $\tilde{H}_b$ in the perturbed flow, $\beta = 0, 1, 2, \ldots$ is the mode of the perturbation, and $C = C_{re} + iC_{im}$ is the eigenvalue of the linear homogeneous problem. From the equations and boundary conditions (2)-(8) above, the complex amplitudes have the form

$$\Phi_{*f}(Y) = A_1 I_\beta(\alpha Y) + A_2 K_\beta(\alpha Y),\quad \Phi_{*i} = A_3 I_\beta(\alpha Y),\quad \Phi_{*e} = A_4 K_\beta(\alpha Y) \tag{9}$$

where I_β, K_β are the modified Bessel functions of the first and second kind of order β. From (9) we can obtain the characteristic equations, which in turn can be used to find C for each of the above problems:

$$m^2n^2 + a_x m^2 + a_y n^2 + a_0 + \rho_i r_i^2(b_x + c_x n^2) + \rho_e r_e^2(b_y + c_y m^2) + \rho_i\rho_e r_i^2 r_e^2 b_0 = 0, \tag{10}$$

$$\rho_i\tau_x r_i^2 + \omega_x - m^2\tau_d' = 0, \tag{11}$$

$$\rho_e\tau_y r_e^2 - \omega_y - n^2\tau_d = 0. \tag{12}$$

Equation (11) can be obtained from (10) by taking the limit $\rho_e \to \infty$ ($\rho_i < \infty$), $\Gamma_e = 0$; Eq. (12) follows from (10) in the limit $\rho_i \to \infty$ ($\rho_e < \infty$), $\Gamma_i = 0$. The coefficients of (10)-(12) are given by

$$a_x = \frac{\omega_y L_x}{L_{xy}}, \quad a_y = -\frac{\omega_x L_y}{L_{xy}}, \quad a_0 = -\frac{\omega_x \omega_y L_0}{L_{xy}},$$

$$b_x = -\frac{\omega_y \tau_x L_0}{L_{xy}}, \quad b_y = \frac{\omega_x \tau_y L_0}{L_{xy}}, \quad b_0 = \frac{\tau_x \tau_y L_0}{L_{xy}} > 0,$$

$$c_x = -\frac{\tau_x L_y}{L_{xy}} > 0, \quad c_y = -\frac{\tau_y L_x}{L_{xy}} > 0,$$

$$L_{xy}(x, y) = I_\beta(y) K_\beta(x) - I_\beta(x) K_\beta(y) > 0,$$

$$L_x(x, y) = I'_\beta(y) K_\beta(x) - I_\beta(x) K'_\beta(y) > 0,$$

$$L_y(x, y) = I_\beta(y) K'_\beta(x) - I'_\beta(x) K_\beta(y) < 0,$$

$$L_0(x, y) = I'_\beta(y) K'_\beta(x) - I'_\beta(x) K'_\beta(y) < 0,$$

$$\tau_x(x) = \frac{I_\beta(x)}{I'_\beta(x)}, \quad \tau_y(y) = \frac{K_\beta(y)}{K'_\beta(y)}, \quad x = \alpha a, \; y = \alpha b,$$

$$m = \alpha U_f + \beta \frac{\Gamma_f}{a^2} - C, \quad n = \alpha U_f + \beta \frac{\Gamma_f}{b^2} - C,$$

$$r_i = \alpha U_i + \beta \frac{\Gamma_i}{a^2} - C, \quad r_e = \alpha U_e + \beta \frac{\Gamma_e}{b^2} - C,$$

$$\omega_x = \alpha \left[\frac{W_a}{a^2} (1 - x^2 - \beta^2) + \frac{\rho_i \Gamma_i^2 - \Gamma_f^2}{a^3} \right],$$

$$\omega_y = \alpha \left[\frac{W_b}{b^2} (1 - y^2 - \beta^2) + \frac{\Gamma_f^2 - \rho_e \Gamma_e^2}{b^3} \right].$$

For the stability of a cylindrical jet in ambient media, condition (4) is replaced by the condition that $\tilde{\Phi}_f$ be bounded as $Y \to 0$, and the boundary condition (6) is applied on the surface of the unperturbed jet $Y = b$. The characteristic equation for this problem has a form similar to (11), (12):

$$\rho_e \tau_y r_e^2 - \omega_y - n^2 \tau_x(y) = 0, \tag{13}$$

where this can be obtained from (12) in the limit $a \to 0$. Equation (10) is an algebraic equation of fourth degree with real coefficients, while (11)-(13) are of second degree in C. The flow will be stable if these equations have real roots and will be unstable for complex roots. The solutions of (11)-(13) can be written in the respective forms:

$$m_{im}^2 = \frac{T_i \gamma_i^2}{(1 + T_i)^2} - \frac{\omega_x}{(1 + T_i)\tau'_d}, \quad m_{re} = \frac{\gamma_i T_i}{1 + T_i}, \tag{14}$$

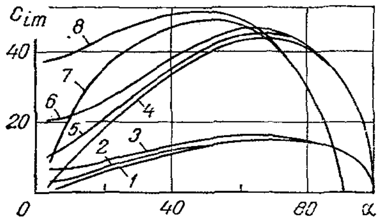
$$T_i = -\rho_i \frac{\tau_x}{\tau'_d}, \quad \tau'_d = \frac{L_x}{L_0} < 0, \; \gamma_i = \alpha \Delta U' + \beta \frac{\Delta \Gamma'}{a^2} \quad (b = 1, \; y = \alpha),$$

$$n_{im}^2 = \frac{T_e \gamma_e^2}{(1 + T_e)^2} + \frac{\omega_y}{(1 + T_e)\tau_d}, \quad n_{re} = \frac{\gamma_e T_e}{1 + T_e}, \tag{15}$$

$$T_e = -\rho_e \frac{\tau_y}{\tau_d}, \quad \tau_d = \frac{L_y}{L_0} > 0, \; \gamma_e = \alpha \Delta U + \beta \frac{\Delta \Gamma}{b^2} \quad (a = 1, \; x = \alpha),$$

$$n_{im}^2 = \frac{T_j \gamma_e^2}{(1 + T_j)^2} + \frac{\omega_y}{(1 + T_j)\tau_x(y)}, \quad T_j = -\rho_e \frac{\tau_y}{\tau_x(y)}. \tag{16}$$

The basic parameters of the problem are: the relative densities of the media ρ_i, ρ_e; discontinuity of the translational velocities $\Delta U' = U_f - U_i$, $\Delta U = U_f - U_e$; torsion parameters Γ_f, Γ_i, Γ_e; Weber numbers W_a, W_b; radii a, b; wave number α; and perturbation mode β. We consider the structure of the above formulas using Eqs. (15) as an example. The square of the imaginary



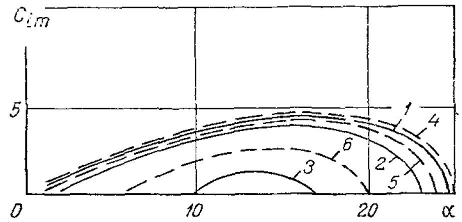


Fig. 2 Fig. 3

Fig. 2. Perturbation growth factors for a layer on the outer surface of a cylinder.

Fig. 3. Perturbation growth factors for a capillary jet.

part n_{im}^2 is made up of two terms. The first corresponds to a Kelvin–Helmholtz instability as a result of the discontinuity of the velocity of the fluid and ambient medium near the interphase [1]. The second term consists of two parts. The first corresponds to a capillary instability; the second corresponds to a Taylor instability and appears as the difference in centrifugal accelerations of the fluid and ambient medium. The structure of m_{im}^2 is analogous. From the above discussion we can make several conclusions on the effect of different physical factors on the stability of a layer on the outer cylinder surface. The velocity discontinuity destabilizes the flow. The relative density ρ_e enhances the Kelvin–Helmholtz instability [7]; however, the first term decreases for large values of ρ_e. The surface tension promotes stability of nonaxisymmetric perturbations $\beta = 1, 2, \ldots$ for any $\alpha \geq 0$, and also axisymmetric perturbations ($\beta = 0$) for $\alpha \geq 1$. Torsion of the outer fluid layer destabilizes the flow; torsion in the ambient medium promotes stability of the perturbations and the effect increases with ρ_e. For a fluid layer on the inner cylinder surface, the above conclusions on the effect of the velocity discontinuity and the surface tension on the flow stability remain in force. But in this case torsion of the fluid stabilizes the flow, and torsion of the medium destabilizes it, with the latter effect increasing with ρ_i.

The results of typical calculations from (15) for the perturbation growth factors in a layer on the outer cylinder surface are shown in Fig. 2. The parameters of the problem are taken to be: $U_i = U_e = U_f = 0$, $\Gamma_i = \Gamma_e = 0$, $\Gamma_f = 10$, $\rho_e = 10^{-2}$, $W_b = 10^{-2}$. Curves 1-3 correspond to a layer thickness 10^{-3} ($a = 1$, $b = 1.001$), curves 4-6 to a layer thickness of 10^{-2} ($a = 1$, $b = 1.01$), and curves 7, 8 to a layer thickness of 10^{-1}. $\beta = 0$ for curves 1, 4, 7; $\beta = 10$ for curves 2, 5; and $\beta = 19$ for curves 3, 6, 8. Interaction with the medium leads to an amplification of the instability of the higher modes. An increase in the fluid layer thickness causes destabilization of the flow and a decrease in the wave number of the fastest growing perturbations; the corresponding flow on the inner surface is stable. In Fig. 3 results of calculations from Eqs. (16) in the case $U_e = 0$, $U_f = 5$, $\Gamma_e = 0$, $\rho_e = 10^{-2}$, $W_b = 10^{-2}$ are shown. Curves 1-3 refer to $\Gamma_f = 0$, curves 4-6 to $\Gamma_f = 0.3$. Curves 1, 2, 3 and 4, 5, 6 correspond to the values $\beta = 0, 5, 9$. For $\Gamma_f = 0$ modes with $\beta \geq 10$ are stable. For $\Gamma_f = 0.3$ modes with $\beta \geq 11$ are stable. Here axisymmetric perturbations are the most unstable. For $U_f = 10$ the most unstable modes are given by $\beta = 1, 2$ and $\alpha \simeq 66$. The data of Fig. 3 show that torsion of the fluid leads to flow destabilization: the maximum perturbation growth factor increases and the instability range expands. Torsion destabilizes the nonaxisymmetric modes to a greater degree.

In the study of the stability of jets we used an algebraic method based on that of Sturm for extracting the real roots [12]. The growth factors of the unstable perturbations are determined from the solution of Eq. (10), which is carried out using the method of Muller [13]. From the four roots we keep the one with the largest imaginary part. Series of calculations are done in which α changes from $\Delta\alpha$ to $N_\alpha \Delta\alpha$ in steps of size $\Delta\alpha$ and $\beta = 1, 2, \ldots, N_\beta$. The results are printed out in tabular form to two significant figures; a typical calculation is shown in Table 1. The calculations show that for nonzero torsions Γ_f, Γ_i, Γ_e and different relative densities (e.g., 0.01 and 1), torsion of the fluid and medium inside the jet cavity destabilizes the flow. Torsions of the medium outside of the jet $\Gamma_e = 1, 2, 3$ lead to stabilization of the flow, but further increase $\Gamma_e = 4, \ldots$ destabilizes the flow. In these cases the nonaxisymmetric modes $\beta > 2$ are the most unstable. Translational flow of the medium outside and (or) inside the jet cavity leads to flow stabilization if it is close to the velocity of the fluid, and leads to flow destabilization in the opposite case. A decrease (from 1 and below) of the relative densities of the ambient media leads to a decrease in the perturbation growth factors. In the thinnest jets these are larger when $\Gamma_e \neq 0$, $\Gamma_i = 0$. If $\Gamma_i \neq 0$, $\Gamma_e = 0$ they are smaller. For a fixed annular layer of ideal fluid in a medium at rest, the

TABLE 1. Perturbation Growth Factors for an Annular Jet

β	α																			
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	3	2	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	4	4	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	5	5	4	3	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
7	6	5	5	4	4	3	2	2	0	0	0	0	0	0	0	0	0	0	0	0
8	7	6	6	5	5	4	4	3	2	1	0	0	0	0	0	0	0	0	0	0
9	7	7	7	6	6	5	5	4	3	2	1	0	0	0	0	0	0	0	0	0
10	8	8	7	7	7	6	6	5	4	4	3	1	0	0	0	0	0	0	0	0
11	9	8	8	8	7	7	6	6	5	5	4	3	1	0	0	0	0	0	0	0
12	9	9	9	8	8	8	7	7	6	6	5	4	3	1	0	0	0	0	0	0
13	10	10	10	9	9	9	8	8	7	7	6	5	4	3	1	0	0	0	0	0
14	11	10	10	10	10	9	9	8	8	7	7	6	5	4	3	0	0	0	0	0
15	11	11	11	11	10	10	10	9	9	8	8	7	6	5	4	3	0	0	0	0
16	12	12	12	11	11	11	10	10	10	9	9	8	7	6	5	4	2	0	0	0
17	13	13	12	12	12	11	11	11	10	10	9	9	8	7	6	5	4	1	0	0
18	13	13	13	13	13	12	12	12	11	11	10	10	9	8	7	6	5	3	0	0
19	14	14	14	13	13	13	13	12	12	11	11	10	10	9	8	7	6	5	3	0

Note: $\rho_i = \rho_e = 10^{-2}$, $W_a = W_b = 10^{-2}$, $a = 0.95$, $b = 1.05$, $\Gamma_i = 0$, $\Gamma_e = 10$, $\Gamma_f = 0$, $U_i = 0$, $U_e = 0$, $U_f = 1$.

stability criteria with respect to infinitesimal perturbations are $x \geqslant 1$ ($\beta = 0$) or $\beta \geqslant 1$ ($x \geqslant 0$), independent of the relative densities $\rho_i = \rho_e$. We also consider the stability criteria in the general case.

We compared the flow stability of circular cross-section capillary jets and jets with an internal cylinder. The analysis showed the stabilizing effect of an internal body on the flow of a jet for both an ideal fluid and a viscous fluid. The viscosity promotes stability of such flows, with the effect increasing as the difference in radii between the jet and internal body decreases ($\beta = 0$, $\rho_e \to 0$).

In the calculations a specialized computational program was created for the modified Bessel functions and their derivatives for values of the argument ranging from 1 to 100 and $\beta = 0, 1, 2, \ldots, 19$. The results coincide with the data of Table 9.11 of [14] to within four significant figures. Our conclusions and results can be used in industrial engineering applications of fluid flow with an interphase for efficient calculation of hydrodynamic instability phenomena.

Finally, we consider the shortwave limits $\alpha \to \infty$. We transform (10) to the form

$$m^2n^2 + \omega_y m^2 + \omega_x n^2 + \omega_x\omega_y + \rho_i r_i^2 (\omega_y + n^2) + \rho_e r_e^2 (\omega_x + m^2) + \rho_i\rho_e r_i^2 r_e^2 = 0. \tag{17}$$

For $\rho_i = \rho_e = \rho_0$ we consider the following cases:

$$\Gamma_f = 0 : m = n, \quad r_i \neq r_e,$$

$$\Gamma_e = \Gamma_i = 0, \quad U_e = U_i : m \neq n, \quad r_i = r_e = r.$$

Then Eq. (17) can be transformed to one of the forms

$$(\rho_0 r_e^2 + \omega_y + m^2)(\rho_0 r_i^2 + \omega_x + m^2) = 0,$$

$$(\rho_0 r^2 + \omega_y + n^2)(\rho_0 r^2 + \omega_x + m^2) = 0.$$

Equation (11) and also Eqs. (12), (13) reduce to

$$\rho_i r_i^2 + \omega_x + m^2 = 0, \quad \rho_e r_e^2 + \omega_y + n^2 = 0.$$

As an example we consider the latter equation. In the limit $\alpha \to \infty$, Eq. (15) takes the form

$$n_{im}^2 = F_\beta(\alpha) = \rho_e \left(\alpha\Delta U + \beta \frac{\Delta\Gamma}{b^2}\right)^2 + \alpha \left(\frac{\Gamma_f^2 - \rho_e\Gamma_e^2}{b^3} - \alpha^2 W_b - \beta^2 \frac{W_b}{b^2}\right). \tag{18}$$

The necessary condition for an extremum in $F_\beta(\alpha)$ is that its derivative vanish:

$$\frac{dF_\beta(\alpha)}{d\alpha}=2\rho_e\Delta U\left(\alpha_*\Delta U+\beta\frac{\Delta\Gamma}{b^2}\right)+\frac{\Gamma_f^2-\rho_e\Gamma_e^2}{b^3}-3W_b\alpha_*^2-\beta^2\frac{W_b}{b^2}=0,\qquad \frac{d^2F_\beta}{d\alpha^2}=2\rho_e(\Delta U)^2-6W_b\alpha_*. \quad (19)$$

The quadratic equation (19) can be solved for $\alpha_*(\beta)$. The solution is particularly simple when $\Delta U = 0$:

$$\alpha_*(\beta)=\sqrt{\frac{1}{3W_b}\left(\frac{\Gamma_f^2-\rho_e\Gamma_e^2}{b^3}-\beta^2\frac{W_b}{b^2}\right)},\quad \frac{d^2F_\beta}{d\alpha^2}<0. \quad (20)$$

The value $\alpha_*(\beta)$ determined from (20) is a maximum of the function n_{im}^2. We note that in the asymptotic limit $\alpha\to\infty$ we have

$$\tau_d=\frac{\exp(y-x)+\exp(x-y)}{\exp(y-x)-\exp(x-y)}\left[1+O\left(\frac{1}{\alpha}\right)\right].$$

For comparison of the asymptotic result with the calculations using (15), recall that for $\alpha\sim 10^2$, the larger the layer thickness $b-1$, the larger $y-x$ and the more precise the asymptotic value $\tau_d\sim 1$.

In actual conditions it is of interest to examine the lengthening of the cavity of a cylindrical annular jet [15]. In order to prolong the cavity, a torsion in the fluid was suggested in [15]. In [16] it was shown that the torsion prevents the closing up of the cavity of a stationary annular jet. We study the stability of an annular jet ignoring the inertia of the ambient media: $\rho_i = \rho_e = 0$. In this case axisymmetric perturbations are the most unstable. Equation (10) is transformed to

$$m^4+(a_x+a_y)\,m^2+a_0=0. \quad (21)$$

The biquadratic equation (21) will have real roots if

$$(a_x+a_y)^2-4a_0>0,\quad a_0>0,\quad a_x+a_y<0. \quad (22)$$

But (22) is known to be satisfied if the following two equivalent inequalities are satisfied:

$$\omega_x<0,\quad \omega_y<0,\quad a_x<0,\quad a_y<0,$$

$$0<\Gamma_a^2(\alpha)=aW_a(1-x^2)<\Gamma_f^2<\Gamma_b^2(\alpha)=bW_b(y^2-1). \quad (23)$$

The torsion of the fluid must be within the limits given by (23) in order not to cause growth of perturbations with wave numbers $\alpha_1\geqslant\alpha$, because $\Gamma_a^2(\alpha_1)\leqslant\Gamma_a^2(\alpha)$, $\Gamma_b^2(\alpha_1)\geqslant\Gamma_b^2(\alpha)$. Actually, perturbations are present in the flow with a range of wave numbers. For Γ_f^2 one can take $\Gamma_a^2(\alpha_m)$, $\Gamma_b^2(\alpha_m)$ for the lower limits of this range. Since the dependence of the roots of the characteristic equation (10) on ρ_i, ρ_e is continuous, in a sufficiently small neighborhood of the state $\rho_i = \rho_e = 0$, $\beta = 0$ there are states with ρ_i, $\rho_e \neq 0$, $\beta = 0$ corresponding to stable flow in an annular jet. This analysis explains the observed lengthening of the cavity of an annular jet moving in air in terms of the torsion of the fluid.

The expression for the growth factor of shortwave perturbations in this case has the form $n_{im} = \sqrt{\omega_y}$ $(\omega_x < 0)$. The wave number and growth factor of the fastest growing perturbation are given by

$$\alpha_*=\sqrt{\frac{\Gamma_f^2}{3b^3W_b}},\quad n_{im}(\alpha_*)=\sqrt{\frac{2\alpha_*\Gamma_f^2}{3b^3}}.$$

Perturbations with $\alpha > \alpha_0 = \sqrt{3}\alpha_*$ are stable. Torsion of the fluid expands the unstable region with respect to α and also causes an increase in the growth factors. The torsion of the fluid can vary between the limits given by the inequality

$$\Gamma_f^2<\Gamma_b^2(\alpha)=\alpha^3b^3W_b, \quad (24)$$

without causing growth of perturbations with wave numbers $\alpha_1\geqslant\alpha$.

NOTATION

a, b, radii of unperturbed free surfaces of an annular jet or fluid layers inside and outside a circular cylinder; $\tilde{H}_a$, $\tilde{H}_b$, perturbed surfaces; W_a, W_b, corresponding Weber numbers; $W = \sigma/(\rho_F U_0^2 R_0)$; σ, surface tension; $\tilde{p}_a$, $\tilde{p}_i$, $\tilde{p}_b$, $\tilde{p}_e$, pressure perturbations in the fluid and ambient media close to interphases; ρ_i, ρ_e, relative densities of the media; X, Y, θ, axial, radial, and angular coordinates; t, time; subscripts X, Y, θ, t denote partial differentiation with respect to the corresponding variable; $\Delta = \frac{\partial^2}{\partial X^2} + \frac{1}{Y}\frac{\partial}{\partial Y}\left(Y\frac{\partial}{\partial Y}\right) + \frac{\partial^2}{\partial \theta^2}$, Laplace operator; $i = \sqrt{-1}$.

LITERATURE CITED

1. G. Birkhoff and E. H. Zarantonello, Jets, Wakes, and Cavities, Academic Press (1957).
2. Yu. F. Dityakin, L. A. Klyachko, B. V. Novikov, and V. I. Yagodkin, Dispersion of Fluids [in Russian], Mashinostroenie (1977).
3. A. S. Lyshevskii, Mechanism of Fluid Fragmentation by Mechanical Spray Pressure [in Russian], Novocherkassk Polytechnical Institute, Novocherkassk (1961).
4. V. M. Entov and A. L. Yarin, "Dynamical equations of liquid drop jets," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5, 11-18 (1980).
5. V. Ya. Shkadov, "Some methods and problems in the theory of hydrodynamic stability," M. V. Lomonosov Moscow State Univ., Inst. of Mechanics, Scientific Works, No. 25 (1973).
6. J. Ponstein, "Instability of rotating cylindrical jets," Appl. Sci. Res. J., Sec. A, 8, No. 6, 425-456 (1959).
7. Yu. M. Shekhtman, "Effect of ambient medium on the stability of jets," Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, No. 11, 1527-1535 (1946).
8. J. W. Hoyt and J. J. Taylor, "Waves on water jets," J. Fluid Mech., 83, 119 (1977).
9. M. S. Uberoi, Chow Chuen-Yen, Narin Jai Prakash, "Stability of coaxial rotating jet and vortex of different densities," Phys. Fluids, 15, 1718 (1972).
10. A. M. Tereshchenko, "Stability of rotational—translational motion of fluids," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3, 191-195 (1971).
11. A. E. Kulago and V. P. Myasnikov, "Calculation of the diameter of a jet under the Taylor-Helmholtz instability," Dokl. Akad. Nauk SSSR, 229, No. 2, 322-324 (1976).
12. A. G. Kurosh, Course of Higher Algebra [in Russian], GTTI, Moscow—Leningrad (1946).
13. V. V. Voevodin, Numerical Methods in Algebra. Theory and Algorithms [in Russian], Nauka, Moscow (1966).
14. M. Abramovits and I. Stigan (eds.), Handbook of Special Functions [in Russian], Nauka, Moscow (1979), Chap. IX.
15. G. A. Baranov, V. A. Glukhikh, I. R. Kirillov, et al., "Apparatus for the study of liquid metal induction MHD-generator and free fluid emission," Magnitn. Gidrodin., No. 4, 104-108 (1974).
16. V. E. Epikhin, "On the form of twisted annular jets of liquid drops," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5, 144-148 (1979).